no well-defined boundaries were seen below 500 Oe. This seems to indicate that the effects of convection currents prevailed over the aligning force of weak fields. It appears reasonable that the critical wall width is comparable with the distance over which the orientation is usually uniform if no field is applied (see Naggiar's result above).

An alignment inversion wall can be stable or metastable if there are boundary restraints preventing it from moving out of the sample or, if it is cylinderlike, from contracting until it vanishes. A wall may also be stabilized if the sample shape does not allow for a decrease in area by migration. Mainly the first factor was presumably responsible for Williams's results. It is interesting to note that similar considerations are used to explain the stability of disinclinations, the frequently observed linear orientation irregularities characteristic of the nematic mesophase. The formation of alignment inversion walls upon applying the magnetic field requires a strongly nonuniform orientation pattern in the fieldless state. The nonuniformity may be produced by the aforementioned convection currents or by the effect of boundaries. Diffuse inversions performed in this way would become sharp walls when the field is applied.

We wish to point out that we do not attempt to explain the optical activity observed by Williams.[1] However, additional experiments[10] seem to lend further support to our view that he saw alignment inversion walls.

I wish to thank Professor J. L. Ericksen for helpful criticism.

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PHASE INCOHERENCE IN THE dc SUPERCONDUCTING TRANSFORMER*

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(Received 8 July 1968)

A model is presented which explains the observed dependence of the coupling between the primary and secondary vortices on the driving current, or primary vortex velocity, in the dc superconducting transformer. The basic idea is that viscous drag effects lead to a slippage between the vortices in the primary film and those in the secondary film for sufficiently large vortex velocities.

The existence of the dc superconducting transformer[1-4] convincingly supports the idea of current-induced vortex motion. In this experiment, two superconducting films sandwich a very thin dielectric layer. One of the metal films is characteristically about twice as thick as the other. The dielectric layer (SiO) electrically insulates the two films. Passing a current through the thicker film (called the primary) results in a force being exerted upon the Abrikosov microstructure[5] established in the film by means of an applied perpendicular magnetic field. As soon as this force is large enough to depin the vortices, they are said to move through the film and a voltage appears across the primary. Since the thinner film (called the secondary) is positioned so close to the primary, the motion of the primary vortices exerts a drag upon the microstructure existing in the secondary inducing this latter array to move also which results in a voltage appearing across the secondary. No current is applied to the secondary. The experiment does not work if either of the two films is in the normal state.

In Giaever's original paper,[1] two of the V_s, V_p vs I_p curves shown exhibit a dependence of the coupling α on the primary current I_p ($\alpha = V_s/V_p$, where V_s and V_p are the voltages in the secondary and primary films, respectively). The purpose of this note is to present new experimental results which further document this effect and to propose a model to explain it.

To exhibit the effect we present in Fig. 1 our results for a typical type-II thin-film transformer consisting of a 4000-Å $In_{92}Pb_{08}$ primary film separated from a 2000-Å $In_{92}Pb_{08}$ secondary film by a 170-Å SiO layer. The coupling α between the films is shown as a function of primary current I_p for various values of the applied (perpendicular) magnetic field. In this particular transformer, coupling was observed only for temperatures in the range $0.77 \lesssim T/T_c \lesssim 1.00$, where T_c is the transition temperature of the films. The current range for which coupling occurs is defined by the boundaries of the shaded region, the lower limit representing the depinning current and the upper limit representing the critical current of the primary film. As seen in the figure the current leads to a gradual decrease in coupling for magnetic fields in excess of about 8 G and the coupling also decreases with increasing magnetic field.

We propose the following explanation for the decrease in coupling with increasing current (increasing primary vortex velocity). Suppose that both the primary and secondary vortices experience a viscous drag force of the form $R\dot{x}$, where $\dot{x}$ is the velocity of the vortex.[6] Assume also that a vortex in the secondary experiences a coupling force F_d from a nearby vortex in the primary, given by the ad hoc relation[7]

$$F_d = A \sin[(2\pi/\delta)(x_1 - x_2)],$$

where x_1 and x_2 are the position coordinates of the centers of the primary and secondary vortices, respectively, and δ is the distance between nearest-neighbor vortices. Steady-state flux flow corresponds to the following:

$$R_2\dot{x}_2 = A \sin[(2\pi/\delta)(x_1 - x_2)] \quad (1)$$

$$R_1\dot{x}_1 = -A \sin[(2\pi/\delta)(x_1 - x_2)] + F_i, \quad (2)$$

where F_i is the force on the primary vortex due to the primary current I_p. From (1) and (2) it is apparent that

$$\langle\dot{x}_2\rangle = \frac{F_i}{R_1 + R_2} - \frac{R_1}{R_1 + R_2}\frac{\delta}{2\pi}\langle\dot{\varphi}\rangle \quad (3)$$

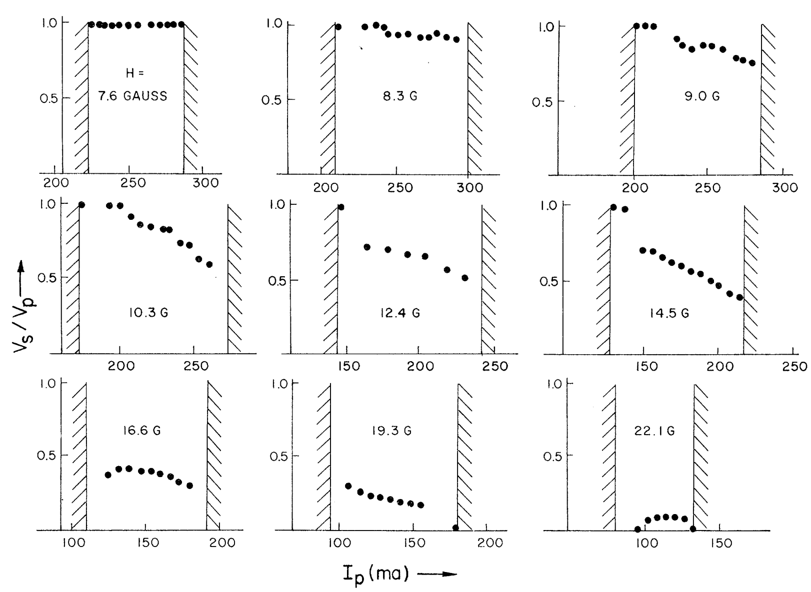

FIG. 1. Coupling (V_s/V_p) in an $In_{92}Pb_{08}$ transformer as a function of primary current I_p for various values of the magnetic field ($T = 3.58$°K). The thickness of the primary is 4000 Å; the secondary, 2000 Å; and the SiO layer, about 170 Å. (For the primary, $T_c = 4.05$°K, and for both films, $\kappa \approx 0.84$.)

and

$$\langle \dot{x}_1\rangle = \frac{F_i}{R_1+R_2} + \frac{R_2}{R_1+R_2}\frac{\delta}{2\pi}\langle\dot{\varphi}\rangle, \quad (4)$$

where $(\delta/2\pi)\varphi = (x_1 - x_2)$. Thus $V_S \propto \langle \dot{x}_2\rangle$ will decrease in the event of a phase slip ($\langle\dot{\varphi}\rangle \neq 0$), whereas $V_p \propto \langle \dot{x}_1\rangle$ will increase; so the coupling $\alpha = \langle \dot{x}_2\rangle/\langle \dot{x}_1\rangle$ will decrease.

From (1) and (2) we find

$$\dot{\varphi} = \xi - \beta\sin\varphi, \quad (5)$$

where $\xi = (2\pi/\delta)F_i/R_1$ and $\beta = A(2\pi/\delta)(R_1+R_2)/R_1R_2$. The quantity ξ reflects the force on a primary vortex arising from the applied current and β reflects the coupling force between the primary and secondary vortices. Integrating Eq. (5) we find that the time T_0 to slip one complete cycle is given by

$$T_0 = 2\pi(\xi^2 - \beta^2)^{-1/2} \quad \text{for} \quad \xi > \beta. \quad (6)$$

This shows that if $\xi > \beta$, an increase in the applied force results in a decrease in T (or increase in $\langle\dot{\varphi}\rangle = 2\pi/T_0$) and a decrease in α [from Eqs. (3) and (4)]. If $\xi \leq \beta$, $\langle\dot{\varphi}\rangle = 0$, and there is no slippage between the primary and secondary vortices.

Increasing the magnetic field leads generally to a decrease in α regardless of the value of I_p or t. The degradation of α with perpendicular magnetic field was first reported by Giaever[4] and is understood in terms of the dependence of the articulation of the periodic field profile on vortex density. There is a flaring of the magnetic field lines at the superconductor-dielectric interface[8] which leads to a degradation of the field profile in the middle of the dielectric layer. If the flaring takes place over a distance which is comparable with or larger than the intervortex spacing δ, the articulation of the periodic field profile is seriously degraded. Consequently, the maximum drag that can be exerted [i.e., the quantity A in Eqs. (1) and (2)] is reduced, leading to $\xi > \beta$, and hence phase slippage ($\langle\dot{\varphi}\rangle > 0$). In the system, $In_{92}Pb_{08}$ (Fig. 1), no coupling was observed for reduced fields $h = H/H_{c2} > 0.2$. In samples made from smaller kappa materials (e.g., Sn) coupling is observed for much larger values of the reduced field and the coupling is not degraded by either high currents or high magnetic fields if the dielectric layer is sufficiently thin.[9] This results presumably from the smaller value of the penetration depth in these materials which ensures a large magnetic field inhomogeneity even for large fields. This enhanced articulation is also reflected in the quantity A of Eqs. (3) and (4). For transformers constructed of low-kappa materials, A is larger which ensures $\xi \lesssim \beta$ or $\langle\dot{\varphi}\rangle = 0$ for the same current density which would lead to phase slippage in a transformer constructed of higher kappa materials.

An explanation alternative to the one given above for the decrease in coupling with primary current is that it is due to the perpendicular component of the magnetic field produced by the primary current. This we showed not to be the important mechanism in the present studies by varying the width of the secondary relative to the width of the primary. Since the field associated with the primary current varies rapidly across the width of the primary, being zero at the center, the degradation of coupling, if it were due to this field, would depend sensitively on the width of the secondary. No such (strong) dependence was observed.

It was found by Sherrill[3] that increasing the ratio d_2/d_1 (where d_2 and d_1 are the thicknesses of the secondary and primary, respectively) leads to a decrease in coupling. This is understood in terms of the above model. From Eqs. (1) and (2) and the above definition of α, one obtains

$$\alpha = \frac{CR_1}{R_2}\left(\frac{F_i}{A\sin\varphi} - 1\right)^{-1}_{\text{av}}. \quad (7)$$

Now, an increase in d_2/d_1 corresponds to a decrease in R_1/R_2 (and therefore a decrease in α), since R is an extrinsic quantity which is directly proportional to the length of the vortex (i.e., d). In the case of pure type-I films, as used in the Sherrill study, the effective screening length depends on film thickness,[10] in which case one must also consider the decrease in the field articulation resulting from a decrease in d_1. This will lead to a decrease in the coupling force A in Eq. (7) and, therefore, a decrease in α.

In this note we have been concerned only with systems which, because of film thickness[11] or kappa value, are believed to exhibit the Abrikosov vortex state. Therefore, we have not discussed or referenced the experiments of Solomon and co-workers[12] on thick type-I magnetically coupled films, which are known to exhibit a more coarsely grained intermediate state structure.

We would like to thank Dr. B. B. Schwartz and Mrs. Jean Parks for interesting discussions.

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*Work supported in part by the Army Research Office

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FERMI SURFACES FOR dhcp La, Nd, AND Pr: RELATIONSHIP TO MAGNETIC ORDERING AND CRYSTAL STRUCTURE*

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(Received 23 September 1968)

The purpose of this Letter is to report the calculation of the relativistic energy bands and Fermi surfaces of the double hexagonal close-packed light rare-earth elements lanthanum, neodymium, and praseodymium. The relationship of the Fermi surface to the ordering of magnetic moments in Nd and Pr and to the occurrence of the double hexagonal close-packed crystal structure is then discussed.

Although the heavier rare earths have been studied extensively in recent years,[1-4] no calculations have yet been made for the double hexagonal close-packed (dhcp) lighter metals.[5] In addition several investigations have been made relating magnetic ordering and Fermi-surface effects in transition and heavy rare-earth metals.[6-8] It was originally proposed by Lomer[6] that the wave vector separating two flat pieces of Fermi surface corresponds to the periodicity of the ordering of the magnetic moments in chromium. Keeton and Loucks[4] have observed that in the heavy rare-earth metals there is good agreement between the magnetic-ordering wave vector and the Fermi-surface separation. This was verified analytically in the recent susceptibility calculations of Evenson and Liu.[8] For Nd and Pr complex magnetic structures have been observed[9,10] which should be related to features of the Fermi surfaces of the two elements.

The energy bands were calculated using the relativistic augmented-plane-wave method developed by Loucks.[11] This method is a relativistic generalization of the augmented-plane-wave (APW) method proposed by Slater[12] and has been previously used for calculation of the electronic structure of the heavy rare-earth elements.

The dhcp unit cell consists of four atoms located at (0, 0, 0), $(\frac{1}{3}, \frac{2}{3}, \frac{1}{4})$, $(0, 0, \frac{1}{2})$, and $(\frac{2}{3}, \frac{1}{3}, \frac{3}{4})$, where (p, q, r) means $p\vec{a}_1 + q\vec{a}_2 + r\vec{a}_3$ and $\vec{a}_1 = a\vec{i}$, $\vec{a}_2 = \frac{1}{2}a\vec{i} + \frac{1}{2}\sqrt{3}a\vec{j}$, and $\vec{a}_3 = c\vec{k}$. The values of the lattice constants a and c used are those given by Pearson.[13] The crystal potential was approximated by a muffin tin potential constructed from a superposition of atomic potentials[14] using the Slater $\rho^{1/3}$ exchange.[15]

The electronic configuration used for La was $5d^1 6s^2$ while for Nd and Pr the configuration was $5d^0 6s^2$, which is the free-ion configuration. While the metallic configuration is actually $5d^1 6s^2$, it has been shown in the heavy rare earths that the difference produces only a small change in the bands.[4] The radius of the APW sphere used was 3.320 a.u. The same 41 reciprocal lattice vectors were used for the wave function expansion at all points in the 1/24 zone. This set of reciprocals was found to give convergence to within 0.002 Ry at the high-symmetry points in the primitive Brillouin zone.

The bands calculated for La along lines of high symmetry are plotted in Fig. 1. The Nd and Pr bands are very similar to those for La and need not be shown at this time.[16] These will be presented in detail in a future paper presenting sus-