

higher than 2.0 mK.

The authors wish to acknowledge very useful conversations with Dr. William C. Thomlinson. They also wish to acknowledge helpful conversations with Professor Herbert Wagner and other members of the theoretical physics group at Jülich. Finally, L.H.N. wishes to thank Professor Wagner and the staff of the Kernforschungsanlage for their very kind hospitality during this stay there.

*Work partially supported by the U. S. Atomic Energy Commission under Contract No. AT(11-1) and by a grant from the Science Research Council of Great Britain.

†Current address: School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton, England. Permanent address: School of Physics and Astronomy, University of Minnesota, Minneapolis, Minn. 55455.

¹D. D. Osheroff, R. C. Richardson, and D. M. Lee, *Phys. Rev. Lett.* **28**, 885 (1972).

²R. A. Scribner, M. F. Panczyk, and E. D. Adams, *J. Low. Temp. Phys.* **1**, 313 (1969).

³R. T. Johnson, O. V. Lounasmaa, R. Rosenbaum, O. G. Symko, and J. C. Wheatley, *J. Low Temp. Phys.* **2**, 403 (1970).

⁴R. A. Guyer, R. C. Richardson, and L. I. Zane, *Rev. Mod. Phys.* **43**, 532 (1971).

⁵W. P. Kirk and E. D. Adams, *Phys. Rev. Lett.* **27**, 392 (1971).

⁶L. I. Zane, *Phys. Rev. Lett.* **28**, 420 (1972).

Nonsingular $S = +1$ Screw Disclination Lines in Nematics

C. Williams,* P. Pierański, and P. E. Cladis

Physique des Solides, †Université Paris-Sud, 91-Orsay, France

(Received 10 April 1972)

We present experimental evidence to support the nonsingular model for screw disclinations of strength $S = +1$.

Constraining a nematic in a long cylinder so that the director has only a radial component (perpendicular to the cylinder axis) results in a splay deformation energy¹ which has a logarithmic divergence as $r \rightarrow 0$ (r being the distance from the cylinder axis). This necessitates the introduction of a core region where the approximation of linear elasticity breaks down. It has recently been shown that the divergence in the splay energy can be avoided by the addition of a finite amount of bend deformation.² The resulting director field has a radial component which decreases as $r \rightarrow 0$ but a z component (parallel to the cylinder axis) which increases as $r \rightarrow 0$. The net energy is then finite, smaller than a pure splay configuration, and does not require a core. Fig. 1(a) shows the theoretical configuration in the case of methoxybenzilidenebutylaniline (MBBA) for which we have taken $K_{11}/K_{33} = 0.7$; K_{33} is the bend elastic constant and K_{11} the splay elastic constant.¹

This Letter gives experimental evidence which supports the nonsingular model.

A thin film of hexadecyltrimethyl ammonium bromide on a clean glass surface orients the director of MBBA molecules perpendicular to the surface. We have treated the inside surface of a

glass capillary tube such that an $S = +1$ disclination line (rotation of 2π of the director along a path coinciding with the boundary, $r = R$) is generated. The tube was then inserted in a layer of optical epoxy resin between two glass plates to avoid optical effects due to its cylindrical shape. The tube was studied with its axis perpendicular

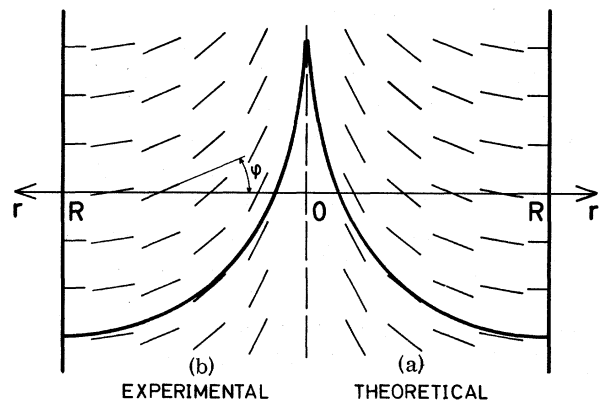
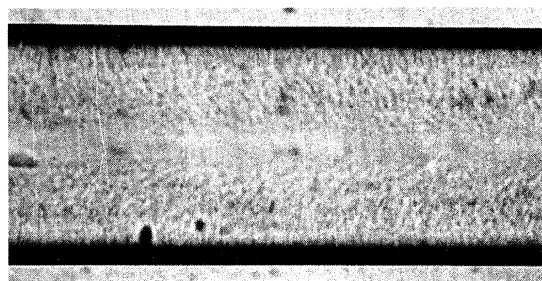
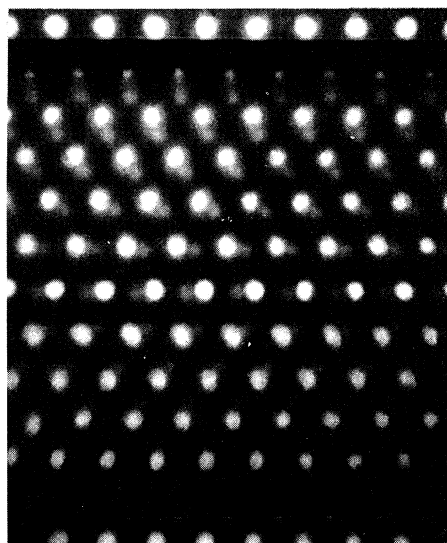


FIG. 1. (a) The molecular orientation around a radial $S = +1$ line deduced from Cladis and Kleman's (Ref. 2) model for MBBA [$K_{33} = 8 \times 10^{-7}$ (cgs), $K_{11}/K_{33} = 0.7$]. (b) Experimentally deduced values for ϕ as found from measuring the inclination of the extraordinary image with respect to the ordinary image.



(a)



(b)

FIG. 2. (a) Focusing slightly above the diametral plane of the tube, an intense fluctuation of the extraordinary ray is seen everywhere except in the middle of the tube. The polarization is perpendicular to tube axis and the analyzer is removed. Turning the polarizer by 90° results in the reversal of this contrast (see text). (b) Image doubling due to the tilt of the director with respect to the incident beam. The fainter dots for $r \neq 0$ correspond to the ordinary image and the brighter ones to the extraordinary image (since the polarizer is 90° to the tube axis). In the center of the tube the intensity relationship is reversed.

to the microscope axis.

We have observed the equilibrium configuration which appears a few minutes after the preparation of the sample. With a polarizing microscope, we can see the flickering of intensity of the light associated with fluctuations of the extraordinary refractive index. If the axis of the tube is perpendicular to the direction of polarization of the

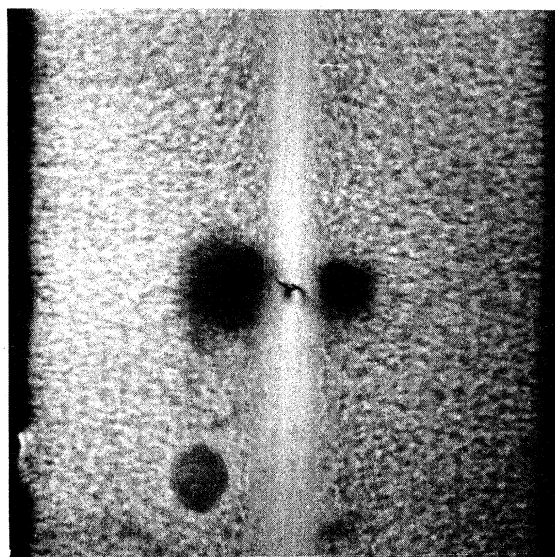
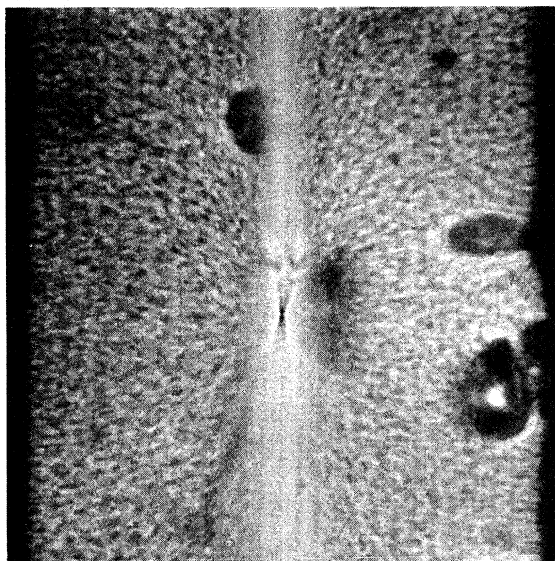


FIG. 3. The singular points shown here lie in the plane perpendicular to the tube axis where the bend angle changes sign (polarizer perpendicular to tube axis, no analyzer). The molecular repartition as deduced from the double-image technique is shown in Fig. 4.

incoming light and we focus on a plane containing this axis (no analyzer), the flickering is visible everywhere except in the center of the tube. The situation is reversed if we rotate the sample by $\pi/2$, i.e., the flickering is now only along the tube axis. Furthermore, it is always highly anisotropic. At $r=0$ it is parallel to the cylinder axis. As $r \rightarrow R$, it curves continuously to become per-

pendicular to the axis at the boundary, $r=R$. It indicates a smooth curvature of the molecular orientation [Fig. 2(a)].

Another way of seeing the molecular orientation is to observe the image of a regular pattern of luminous dots through the tube. Each dot gives rise to an ordinary image and an extraordinary image displaced with respect to one another by an observable distance.³ In our geometry the doubling of the dots is minimal when focusing on a plane close to the medium plane of the specimen. Raising the focus from that position results in the appearance of two distinct sets of dots [Fig. 2(b)]. The inclination of the displacement is a measure of the bend component of the director in the plane of observation. It is to be noticed that the anisotropy of the intensity fluctuations is directed along the displacement direction of the extraordinary image. We have measured the inclination of the displacement as a function of the ratio r/R and have compared this angle with the theoretically computed value² ψ (inclination of the molecules on a diametral plane) using the expression

$$\frac{r}{R} = \frac{\cos k \sin \varphi - (1 - \sin^2 k \sin^2 \varphi)^{1/2}}{\cos k \sin \varphi + (1 - \sin^2 k \sin^2 \varphi)^{1/2}} \times \exp(-\psi \tan k),$$

where $\sin \psi = \sin k \sin \varphi$ and $\tan^2 k = (K_{33} - K_{11})/K_{11}$. Even without correcting for the optical geometry, the agreement with theory is reasonable.

We have frequently observed singular points appearing along the tube axis. These occur when the bend direction changes (see Fig. 3) and are of two types which alternate along the line. Two consecutive points can annihilate leaving no trace. The projections of the molecular orientation on the observation plane, as deduced from the displacement of images, are shown in Fig. 4. Varying R from 20 to 200 μm has not led to any qualitative difference in these observations.

In summary, these results prove definitely that the planar solution for the $S=+1$ disclination is not an equilibrium configuration and that the director has a component out of the plane which increases monotonically from the outside boundary to the center of the tube at $r=0$. The results are consistent with the nonsingular model but do not enable one to rule out the existence of a core of

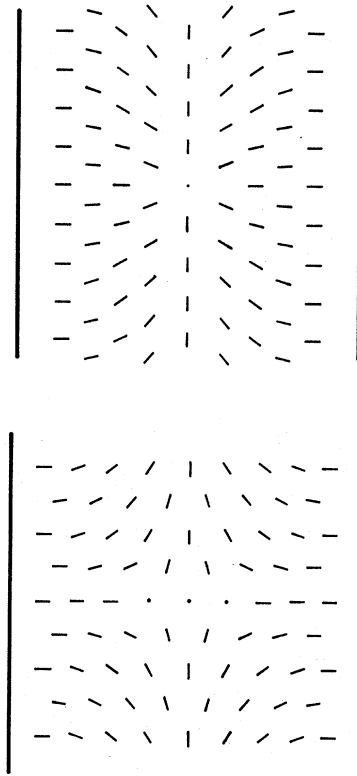


FIG. 4. Molecular inclination as found by the method of double images around a singular point for two cases we have observed.

radius smaller than a few micrometers.

We wish to thank Dr. Kléman for stimulating discussions.

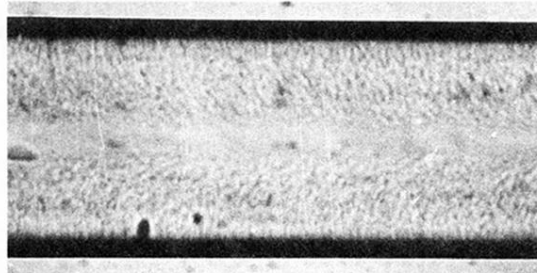
*Permanent address: Service de Physique du Solide, Centre d'Etudes Nucléaires, Saclay, France.

†Laboratoire associé au Centre National de la Recherche Scientifique.

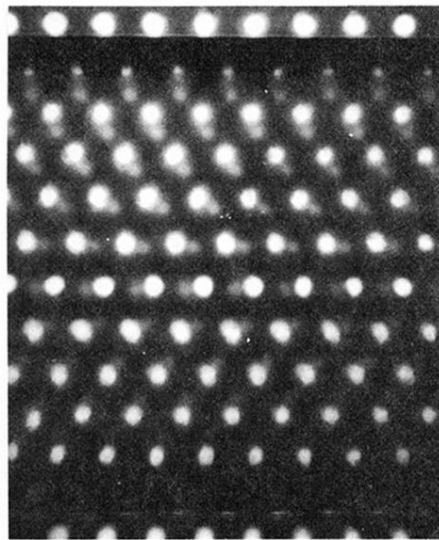
¹F. C. Frank, *Discuss. Faraday Soc.* **25**, 1 (1958).

²P. E. Cladis and M. Kléman, to be published. We have recently been made aware of a similar calculation by R. B. Meyer (private communication) restricted to the case of isotropic elasticity. Meyer has also studied the $S=+1$ lines in a capillary tube. The present paper supplements his observations by the use of the imaging technique described in this paper and by the observation of singular points along the cylinder axis.

³See for example, N. H. Hartshorne and A. Stuart, *Crystals and the Polarizing Microscope* (Edward Arnold and Company, London, 1970), 4th ed.



(a)



(b)

FIG. 2. (a) Focusing slightly above the diametral plane of the tube, an intense fluctuation of the extraordinary ray is seen everywhere except in the middle of the tube. The polarization is perpendicular to tube axis and the analyzer is removed. Turning the polarizer by 90° results in the reversal of this contrast (see text). (b) Image doubling due to the tilt of the director with respect to the incident beam. The fainter dots for $r \neq 0$ correspond to the ordinary image and the brighter ones to the extraordinary image (since the polarizer is 90° to the tube axis). In the center of the tube the intensity relationship is reversed.

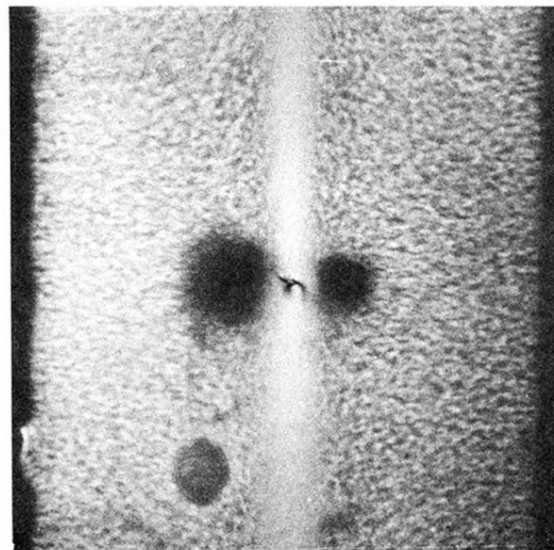
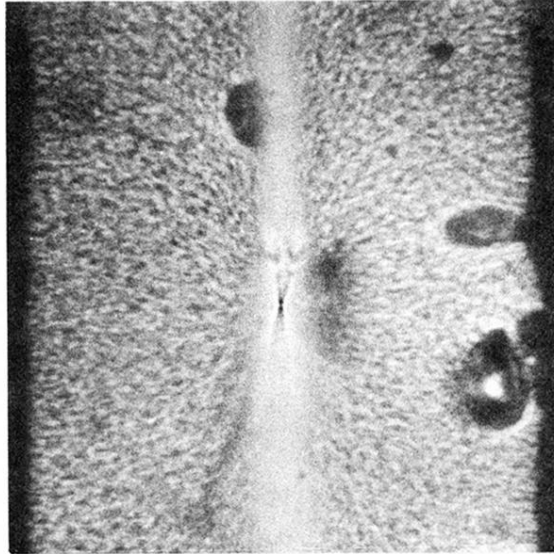


FIG. 3. The singular points shown here lie in the plane perpendicular to the tube axis where the bend angle changes sign (polarizer perpendicular to tube axis, no analyzer). The molecular repartition as deduced from the double-image technique is shown in Fig. 4.